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# MIXED CONVECTION ABOUT A NON-ISOTHERMAL VERTICAL SURFACE IN A POROUS MEDIUM

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## SUMMARY

The problem of mixed convection about non-isothermal vertical surfaces in a saturated porous medium is analysed using boundary layer approximations. The analysis is made assuming that the surface temperature varies as an arbitrary function of the distance from the origin. A perturbation technique has been applied to obtain the solutions. Using the differentials of the wall temperature, which are functions of distance along the surface, as perturbation elements, universal functions are derived for various values of the governing parameter $Gr/Re$. Both aiding and opposing flows are considered. The universal functions obtained can be used to estimate the heat transfer and fluid velocity inside the boundary layer for any type of wall temperature variation. As a demonstration of the method, heat transfer results have been presented for the case of the wall temperature varying as a power function of the distance from the origin. The results have been studied for various combinations of the parameters $Gr/Re$ and the power index $m$, taking both aiding and opposing flows into consideration. On comparing these results with those obtained by a similarity analysis, the agreement is found to be good.

KEY WORDS Porous medium Mixed convection Nonisothermal vertical surface Boundary layer type analysis Perturbation technique

## 1. INTRODUCTION

Transport processes in porous media occur in many different fields and engineering applications, such as petroleum reservoirs and geothermal operations, packed-bed chemical reactors, grain storage, insulation, and cores of nuclear reactors. This has led to extensive research into the subject for the past two decades.

The study of mixed convection in a porous medium has important applications in geothermal reservoirs, where fluid motion may take place due to pressure gradients generated as a result of withdrawal or injection of geothermal fluids. Such fluid motion, when combined with the buoyancy force due to heating or cooling from a vertical surface, will give rise to mixed convection adjacent to the vertical surface. The first work in this connection was done by Combarnous and Bia[1] who studied the effect of mean flow on the onset of stability in a porous medium bounded by two isothermal parallel plates. Later Cheng[2] considered the problem of mixed convection about a wedge and obtained a similarity solution for the special case where the free stream velocity and wall temperature vary according to some power function of distance. Cheng and Minkowycz[3] have analysed free convection about a vertical plate embedded in porous media.

A study of mixed convection about a horizontal surface embedded in a porous medium where the gravitational force acts perpendicular to the surface has been made by Cheng.[4] He obtained similarity solutions for the case of zero angle of attack with constant heat flux and for stagnation

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point flows with $T_w \propto x^2$. Later Minkowycz *et al.*[5] analysed non-similar boundary layers in mixed convection about a horizontal heated surface. Bejan,[7,8] Walker and Homsay,[9] Poulikakos and Bejan,[10] Prasad and Kulacki[11] and Tong and Subramanian[12] have investigated the various aspects of studies in porous media.

In the present paper, a boundary layer analysis has been done for mixed convection about non-isothermal vertical surfaces in a porous medium. (Cheng[2] has already analysed the same for isothermal vertical surfaces.) It is assumed that the surface temperature varies as an arbitrary function of the distance from the origin. The solutions, which are based on perturbations to the isothermal case, can be used to obtain heat transfer results for any type of wall temperature variation.

## 2. ANALYSIS

We consider the problem of combined free and forced convection in a fluid-saturated porous medium adjacent to a vertical impermeable wall. The temperature of the vertical wall is assumed to vary as an arbitrary function of the distance from the origin. Figures 1(a) and (b) show the physical model and co-ordinate system, where $x$ and $y$ are the Cartesian co-ordinates in the directions perpendicular to and along the vertical wall. The origin of the co-ordinate system is chosen at that point where the wall temperature deviates from that of the mainstream fluid. We shall assume that:

(i) there is no phase change occurring in the fluid;
(ii) the convective fluid and the porous medium are everywhere in local thermodynamic equilibrium;
(iii) the properties of the fluid and the porous medium are homogeneous and isotropic;
(iv) the Boussinesq approximation can be applied.

Under these assumptions the governing equations of the problems are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u=-\frac{K}{\mu}\frac{\partial p}{\partial x}, \tag{2}$$

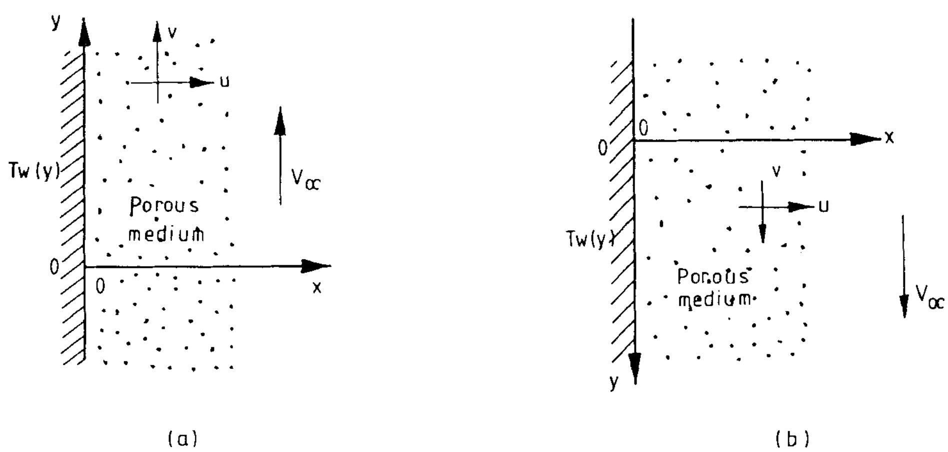


Figure 1. Physical model and co-ordinate systems

$$v=-\frac{K}{\mu}\left(\frac{\partial p}{\partial y}\pm\rho g\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

$$\rho=\rho_\infty[1-\beta(T-T_\infty)]. \tag{5}$$

The boundary conditions are

$$x=0, \qquad u=0, \qquad T_w=T_w(y), \tag{6}$$

$$x\to\infty, \qquad v\to V_\infty, \qquad T\to T_\infty. \tag{7}$$

The '+' and '−' signs in equation (3) indicate aiding and opposing flows as shown in Figures 1(a) and (b) respectively. The wall temperature is assumed to be higher than that of the surrounding fluid. In aiding flows the free stream velocity is in the same direction as the buoyancy force, while in opposing flows they are in opposite directions. The free stream velocity $V_\infty$ is assumed to be uniform.

The continuity equation (1) can be satisfied automatically by introducing the stream function $\psi$ as

$$u=\partial\psi/\partial y, \qquad v=-\partial\psi/\partial x. \tag{8}$$

Eliminating $p$ from equations (2) and (3) by cross-differentiation and making use of equation (5), the resulting equation in terms of $\psi$ is

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=\mp\frac{K\beta g}{\nu}\frac{\partial T}{\partial x}, \tag{9}$$

where the '−' and '+' signs indicate aiding and opposing flows respectively.

In terms of $\psi$ the energy equation (4) can be written as

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{10}$$

If convection takes place in a thin layer such that $\partial/\partial y\ll\partial/\partial x$, we have $\partial^2\psi/\partial y^2\ll\partial^2\psi/\partial x^2$ in equation (9) and $\partial^2 T/\partial y^2\ll\partial^2 T/\partial x^2$ in equation (10). Neglecting the smaller orders in equations (9) and (10), the simplified forms of the governing equations are

$$\frac{\partial^2\psi}{\partial x^2}=\mp\frac{K\beta g}{\nu}\frac{\partial T}{\partial x}, \tag{11}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}. \tag{12}$$

The boundary conditions are

$$\partial\psi/\partial y=0, \qquad T=T_w(y) \quad \text{at } x=0, \tag{13}$$

$$\partial\psi/\partial x\to V_\infty, \qquad T\to T_\infty \qquad \text{as } x\to\infty. \tag{14}$$

*Solution of equations*

The governing equations (11) and (12) must now be suitably transformed so as to seek a perturbation type of solution. We introduce a new independent variable $\eta$, a non-dimensional stream function $f$ and a non-dimensional temperature $\theta$. Both $f$ and $\theta$ are assumed to be functions of $\eta$ and a set of infinite number of variables $\{\lambda_n(y)\}$, which together express the characteristics of the boundary layer.

Let the function $f$ be related to the original stream function by the equation

$$f(\eta, \lambda_0, \lambda_1, \ldots, \lambda_n, \ldots) = \psi(x, y)/(\alpha V_\infty y)^{1/2}. \tag{15}$$

Similarly let $\theta$ be defined as

$$\theta(\eta, \lambda_0, \lambda_1, \ldots, \lambda_n, \ldots) = \frac{T(x, y) - T_\infty}{T_w(y) - T_\infty}, \tag{16}$$

$$\eta = \left(\frac{V_\infty y}{\alpha}\right)^{1/2} \frac{x}{y}. \tag{17}$$

The set $\{\lambda_n(y)\}$ are as yet undetermined functions of $y$.

Carrying out the above transformations in equations (11) and (12), the resulting equations in terms of $f$ and $\theta$ are

$$\frac{\partial^2 f}{\partial \eta^2} = \mp \frac{Gr}{Re}\frac{\partial \theta}{\partial \eta}, \tag{18}$$

$$\frac{f}{2}\frac{\partial \theta}{\partial \eta} + \frac{\partial \theta}{\partial \eta} y \sum_{n=0}^{\infty} \frac{\partial f}{\partial \lambda_n}\frac{\partial \lambda_n}{\partial y} - \frac{\partial f}{\partial \eta} y \sum_{n=0}^{\infty} \frac{\partial \theta}{\partial \lambda_n}\frac{\partial \lambda_n}{\partial y} - \frac{\partial f}{\partial \eta}\theta y \frac{(\mathrm{d}/\mathrm{d}y)[T_w(y)]}{T_w - T_\infty} = \frac{\partial^2 \theta}{\partial \eta^2}, \tag{19}$$

where $Gr$ is the modified Grashof number defined as

$$Gr = K\beta g(T_w - T_\infty)y/v^2 \tag{20}$$

and $Re$ is the Reynolds number defined as

$$Re = V_\infty y/v. \tag{21}$$

Now a study of equation (19) reveals that it becomes a function of $\eta$ and $\lambda_n(y)$ when $\lambda_n(y)$ is defined as

$$\lambda_n = \frac{y^{n+1}}{T_w - T_\infty}\frac{\mathrm{d}^{n+1}}{\mathrm{d}y^{n+1}}(T_w - T_\infty). \tag{22}$$

Here it is assumed that $T_w - T_\infty$ is infinitely differentiable with respect to $y$.

Even after substituting equation (22) into equation (19), the resulting equation is still a partial differential equation. To transform it into an ordinary differential equation, $f$ and $\theta$ are expanded as power series of $\lambda_n$ with functions of $\eta$ as coefficients:

$$f(\eta, \lambda_0, \lambda_1, \ldots, \lambda_n, \ldots) = F(\eta) + \lambda_0 f_0(\eta) + \lambda_1 f_1(\eta) + \ldots + \lambda_0^2 f_{00}(\eta) + \lambda_1^2 f_{11}(\eta) + \ldots, \tag{23}$$

$$\theta(\eta, \lambda_0, \lambda_1, \ldots, \lambda_n, \ldots) = H(\eta) + \lambda_0 \theta_0(\eta) + \lambda_1 \theta_1(\eta) + \ldots + \lambda_0^2 \theta_{00}(\eta) + \lambda_1^2 \theta_{11}(\eta) + \ldots. \tag{24}$$

Using the transformations (22), (23) and (24) in equations (18) and (19), and collecting terms of equal $\lambda_n$, we get an infinite number of sets of ordinary differential equations, the first few of which are as follows:

$$F'' = \mp(Gr/Re)H',$$
$$\tfrac{1}{2}FH' = H'', \tag{25}$$

$$f_0'' = \mp(Gr/Re)\theta_0',$$
$$\tfrac{1}{2}F\theta_0' + \tfrac{3}{2}f_0 H' - F'\theta_0 - F'H = \theta_0'', \tag{26}$$

$$f_1'' = \mp(Gr/Re)\theta_1',$$
$$\tfrac{1}{2}F\theta_1' + \tfrac{5}{2}f_1H' - 2F'\theta_1 + H'f_0 - F'\theta_0 = \theta_1'', \quad (27)$$
$$f_{00}'' = \mp(Gr/Re)\theta_{00}',$$
$$\tfrac{1}{2}F\theta_{00}' + \tfrac{5}{2}f_{00}H' + \tfrac{3}{2}f_0\theta_0' - H'f_0 - 2F'\theta_{00} - f_0'\theta_0 - f_0'H = \theta_{00}''. \quad (28)$$

The boundary conditions of the above equations are as follows:

$$F = f_0 = f_1 = f_{00} = f_2 = 0,$$
$$H = 1, \qquad \theta_0 = \theta_1 = \theta_{00} = \theta_2 = 0 \text{ at } \eta = 0, \quad (29)$$
$$F' = -1, \qquad f_0' = f_1' = f_{00}' = f_2' = 0,$$
$$H = \theta_0 = \theta_1 = \theta_{00} = \theta_2 = 0 \quad \text{as} \quad \eta \to \infty. \quad (30)$$

In all the above equations a prime indicates differentiation with respect to $\eta$.

## 3. RESULTS AND DISCUSSION

Equations (25)–(28) with boundary conditions (29) and (30) were solved using the fourth-order Runge–Kutta method. Universal functions were obtained for both aiding and opposing flows with $Gr/Re$ as a parameter. The first few universal functions required to calculate the temperature profiles are shown in Figures 2–6. The corresponding universal constants for aiding and opposing flows are shown in Tables I and II respectively. It may be noted that the universal function obtained by solving the equation set (25) is actually the isothermal solution obtained by Cheng.[2]

The result of greatest practical importance is the heat transfer rate. The local surface heat flux along the vertical surface may be expressed as

$$q = -k_m(\mathrm{d}T/\partial x)_{x=0}. \quad (31)$$

Expressing $q$ in terms of $\eta$ and $\theta$,

$$q = -k_m(T_w - T_\infty)(V_\infty/\alpha y)^{1/2}(\partial\theta/\partial\eta)_{n=0}. \quad (32)$$

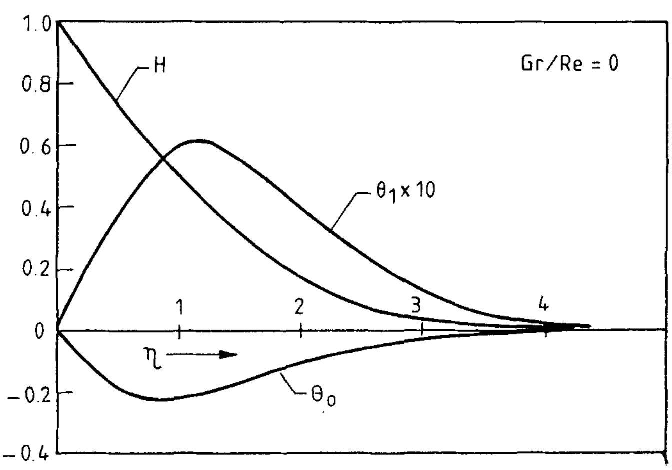


Figure 2. Universal functions of temperature

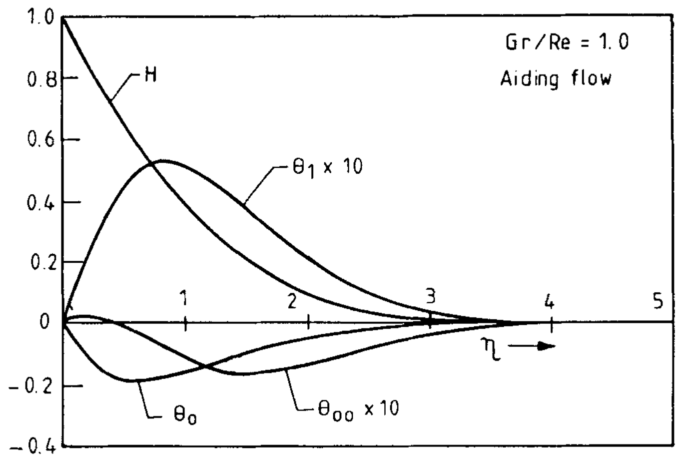


Figure 3. Universal functions of temperature

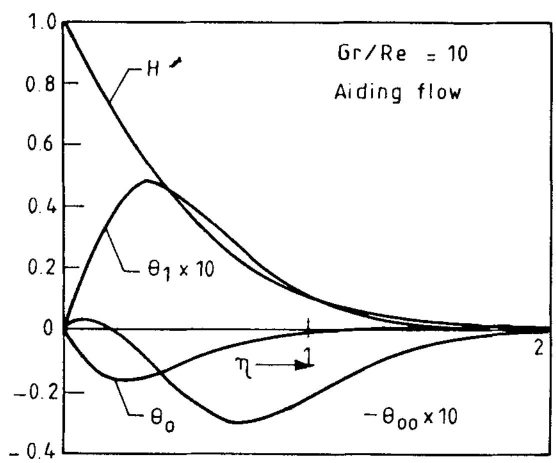


Figure 4. Universal functions of temperature

The local heat transfer coefficient is expressed as

$$h = q/(T_w - T_\infty) \tag{33}$$

and the local Nusselt number is

$$Nu = hy/k_m. \tag{34}$$

Combining equations (32), (33) and (34), we can express the dimensionless heat transfer rate as

$$Nu/(Re\,Pr)^{1/2} = -(\partial\theta/\partial n)_{n=0}. \tag{35}$$

Equation (35) can be thrown into perturbation form by using equation (24):

$$Nu/(Re\,Pr)^{1/2} = -[H'(0) + \lambda_0\theta_0'(0) + \lambda_1\theta_1'(0) + \ldots + \lambda_0^2\theta_{00}'(0) + \ldots]. \tag{36}$$

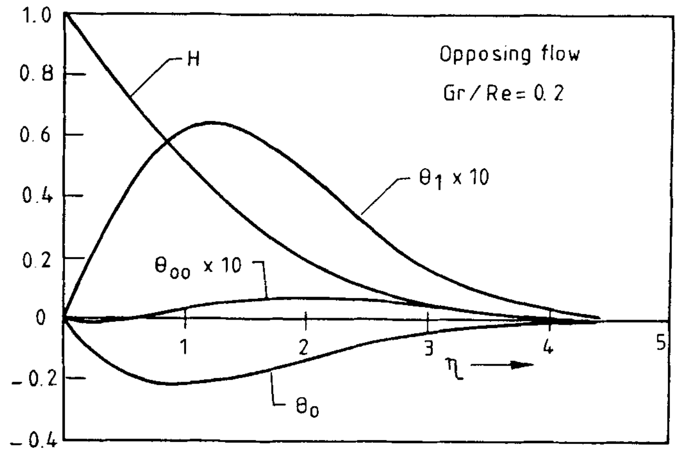


Figure 5. Universal functions of temperature

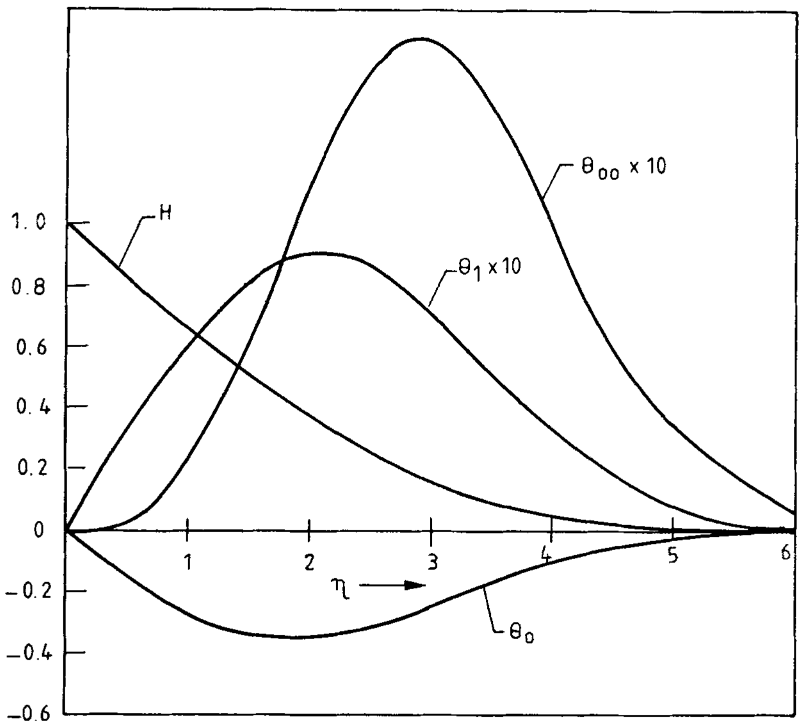


Figure 6. Universal functions of temperature; opposing flow, $Gr/Re = 1{\cdot}0$

Using the universal constants of Tables I and II and the heat transfer formula of equation (36), heat transfer results can easily be calculated for any type of wall temperature function. Heat transfer results for a power-law type of wall temperature variation are presented below.

Table I. Universal constants for aiding flows

| $Gr/Re$ | $H'(0)$ | $\theta'_0(0)$ | $\theta'_1(0)$ | $\theta'_{00}(0)$ | $F'(0)$ | $f'_0(0)$ | $f'_1(0)$ | $f'_{00}(0)$ |
|---|---|---|---|---|---|---|---|---|
| 0·0 | −0·5642 | −0·5758 | 0·0941 | 0·0 | −1·0 | 0·0 | 0·0 | 0·0 |
| 0·5 | −0·6474 | −0·6733 | 0·1053 | 0·0053 | −1·5 | 0·0 | 0·0 | 0·0 |
| 1·0 | −0·7205 | −0·7595 | 0·1151 | 0·0095 | −2·0 | 0·0 | 0·0 | 0·0 |
| 3·0 | −0·9572 | −1·0408 | 0·1471 | 0·0209 | −4·0 | 0·0 | 0·0 | 0·0 |
| 10·0 | −1·5155 | −1·7209 | 0·2208 | 0·0403 | −11·0 | 0·0 | 0·0 | 0·0 |
| 20·0 | −2·0652 | −2·4159 | 0·2887 | 0·0528 | −21·0 | 0·0 | 0·0 | 0·0 |

Table II. Universal constants for opposing flows

| $Gr/Re$ | $H'(0)$ | $\theta'_0(0)$ | $\theta'_1(0)$ | $\theta'_{00}(0)$ | $F'(0)$ | $f'(0)$ | $f'_1(0)$ | $f'_{00}(0)$ |
|---|---|---|---|---|---|---|---|---|
| 0·2 | −0·5269 | −0·5325 | 0·0892 | −0·00254 | −0·8 | 0·0 | 0·0 | 0·0 |
| 0·4 | −0·4866 | −0·4859 | 0·0839 | −0·00538 | −0·6 | 0·0 | 0·0 | 0·0 |
| 0·6 | −0·4421 | −0·4351 | 0·0781 | −0·00835 | −0·4 | 0·0 | 0·0 | 0·0 |
| 0·8 | −0·3917 | −0·3792 | 0·0717 | −0·01068 | −0·2 | 0·0 | 0·0 | 0·0 |
| 1·0 | −0·3321 | −0·3180 | 0·0637 | −0·00618 | 0·0 | 0·0 | 0·0 | 0·0 |

Table III. Values of $Nu/(Re\,Pr)^{1/2}$ for aiding flows by perturbation method (wall temperature varying as power function)

| | $m$ | | | | |
|---|---|---|---|---|---|
| $Gr/Re$ | 0·0 | 0·25 | 0·50 | 0·75 | 1·0 |
| 0·0 | 0·5642 | 0·7258 | 0·8756 | 1·0137 | 1·14 |
| 0·5 | 0·6474 | 0·8352 | 1·0091 | 1·1690 | 1·3154 |
| 1.0 | 0·7205 | 0·9314 | 1·126 | 1·306 | 1·4705 |
| 3·0 | 0·9572 | 1·2437 | 1·5092 | 1·7566 | 1·9771 |
| 10·0 | 1·5155 | 1·9846 | 2·4211 | 2·8249 | 3·1961 |
| 20·0 | 2·0652 | 2·7200 | 3·332 | 3·9015 | 4·4283 |

Table IV. Values of $Nu/(Re\,Pr)^{1/2}$ for aiding flows by similarity analysis (wall temperature varying as power function)

| | $m$ | | | | |
|---|---|---|---|---|---|
| $Gr/Re$ | 0·0 | 0·25 | 0·50 | 0·75 | 1·0 |
| 0·0 | 0·5642 | 0·7399 | 0·8863 | 1·0179 | 1·1284 |
| 0·5 | 0·6474 | 0·8502 | 1·0191 | 1·1626 | 1·2990 |
| 1·0 | 0·7205 | 0·9473 | 1·126 | 1·3012 | 1·4492 |
| 3·0 | 0·9572 | 1·2597 | 1·5158 | 1·7325 | 1·9319 |
| 10·0 | 1·5155 | 1·9961 | 2·4081 | 2·7618 | 3·262 |
| 20·0 | 2·0652 | 2·7274 | 3·2853 | 3·9265 | 4·322 |

*Power-law variation of wall temperature*

The wall temperature may be assumed to vary as

$$T_w - T_\infty = My^m,$$

where $M$ and $m$ are constants.

For this case the values of $\lambda_0$, $\lambda_1$, etc. may be calculated using equation (22) and are found to be

$$\lambda_0 = m, \qquad \lambda_1 = m(m-1), \qquad \lambda_0^2 = m^2, \qquad \ldots \tag{37}$$

Calculating $\lambda_0$, $\lambda_1$, etc. for various $m$ and making use of Tables I and II, $Nu/(Re\,Pr)^{1/2}$ can be calculated from equation (36) for various values of $m$ and $Gr/Re$. The values are shown in Tables III and V. Tables IV and VI show the heat transfer results obtained by Dutta and Seetharamu[6] by similarity analysis. There is good agreement between the results obtained by the two methods.

When $m=0$ we have the isothermal wall case and the values of $Nu/(Re\,Pr)^{1/2}$ in the above tables correspond to those obtained by Cheng.[2]

The effect of the power index $m$ on the heat transfer rate $Nu/(Re\,Pr)^{1/2}$ for various values of $Gr/Re$ is shown in Figures 7 and 8 for aiding and opposing flows respectively. As expected, the heat transfer rate increases with increasing $m$. It is also observed that for aiding flows the effect of $m$ on the heat transfer rate is more significant at higher values of $Gr/Re$, while for opposing flows the effect is more significant at lower values of $Gr/Re$.

For a given value of $m$, say 0·5, the heat transfer rate is 0·8161 for $Gr/Re = 1{\cdot}0$. As $m$ increases, the increase in the absolute value of $Nu/(Re\,Pr)^{1/2}$ is greater for smaller values of $Gr/Re$ (say $Gr/Re = 0{\cdot}2$) than for larger values of $Gr/Re$ (say $Gr/Re = 1$). In other words, the slope of the heat

Table V. Values of $Nu/(Re\,Pr)^{1/2}$ for opposing flows by perturbation method (wall temperature varying as power function)

| | $m$ | | | | |
|---|---|---|---|---|---|
| $Gr/Re$ | 0·0 | 0·25 | 0·50 | 0·75 | 1·0 |
| 0·2 | 0·5269 | 0·6769 | 0·8161 | 0·9444 | 1·0619 |
| 0·4 | 0·4866 | 0·6241 | 0·7519 | 0·8698 | 0·9779 |
| 0·6 | 0·4421 | 0·5660 | 0·6813 | 0·7877 | 0·8855 |
| 0·8 | 0·3917 | 0·5006 | 0·6019 | 0·6955 | 0·7816 |
| 1·0 | 0·3321 | 0·4239 | 0·5086 | 0·5860 | 0·6563 |

Table VI. Values of $Nu/(Re\,Pr)^{1/2}$ for opposing flows by similarity analysis (wall temperature varying as power function)

| | $m$ | | | | |
|---|---|---|---|---|---|
| $Gr/Re$ | 0·0 | 0·25 | 0·50 | 0·75 | 1·0 |
| 0·2 | 0·5269 | 0·6962 | 0·8292 | 0·9469 | 1·0529 |
| 0·4 | 0·4866 | 0·6376 | 0·7668 | 0·8746 | 0·9718 |
| 0·6 | 0·4421 | 0·5795 | 0·6950 | 0·7970 | 0·8843 |
| 0·8 | 0·3917 | 0·5152 | 0·6135 | 0·7050 | 0·7796 |
| 1·0 | 0·3321 | 0·4378 | 0·5248 | 0·5923 | 0·6648 |

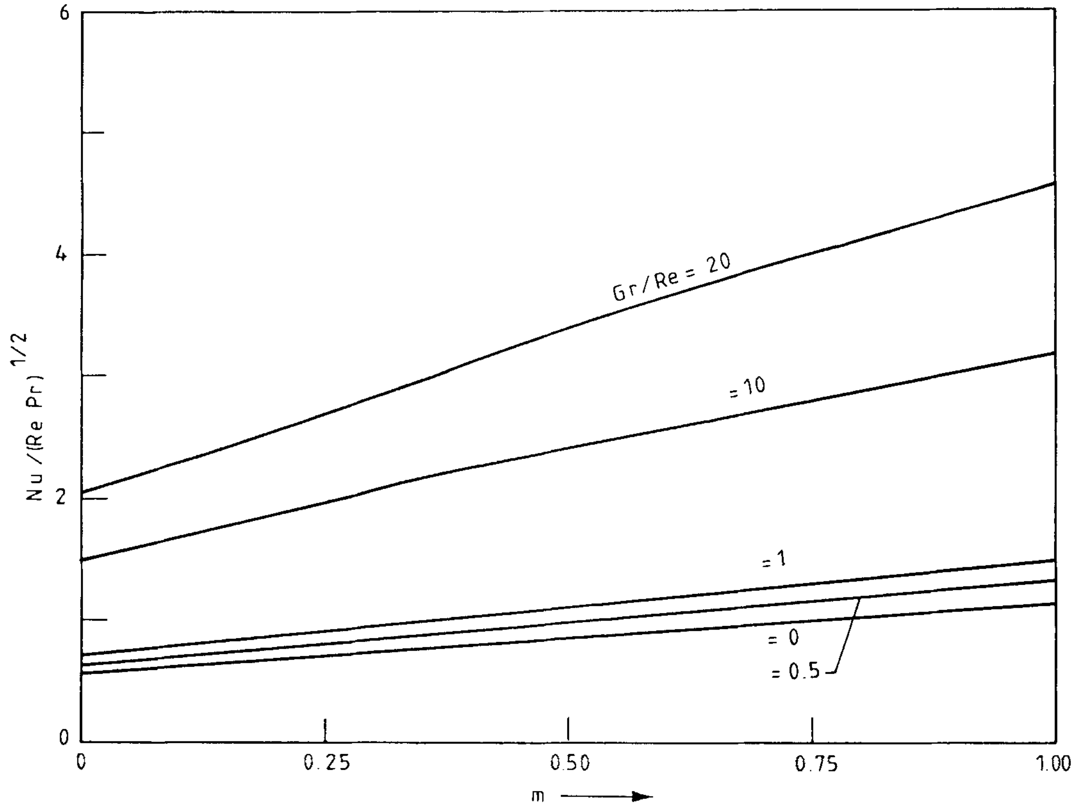


Figure 7. Effect of $m$ on $Nu/(Re\,Pr)^{1/2}$; aiding flow

transfer rate curves with smaller values of $Gr/Re$ is greater than the slope of those with larger values of $Gr/Re$.

From the results obtained by Cheng and Minkowycz[3] for free convection about a vertical plate, we can write down the following expressions for the heat transfer rate for pure free convection for various values of the parameter $m$:

$$Nu/(Re\,Pr)^{1/2} = 0{\cdot}444\,(Gr/Re)^{1/2} \quad (\text{for } m=0), \tag{38}$$

$$Nu/(Re\,Pr)^{1/2} = 0{\cdot}7615\,(Gr/Re)^{1/2} \quad (\text{for } m=0{\cdot}5), \tag{39}$$

$$Nu/(Re\,Pr)^{1/2} = 1{\cdot}001\,(Gr/Re)^{1/2} \quad (\text{for } m=1). \tag{40}$$

The corresponding forced convection results can be obtained from Table III using $Gr/Re=0$.

The heat transfer results have been plotted in Figures 9 and 10 along with the free and forced convection asymptotes. It is observed that for $m=0$ the maximum deviation from the asymptotes occurs near $Gr/Re=1{\cdot}6$. But as the parameter $m$ increases, the point of maximum deviation shifts to lower $Gr/Re$. (The point shifts approximately to $Gr/Re=1{\cdot}35$ for $m=0.5$ and to $Gr/Re=1{\cdot}25$ for $m=1$. The corresponding deviations are about 40%, 37% and 34% for $m=0$, 0·5 and 1 respectively.)

As the parameter $Gr/Re$ increases beyond the point of maximum deviation, the free convection effect dominates and the situation approaches that of pure free convection. This happens at a lower

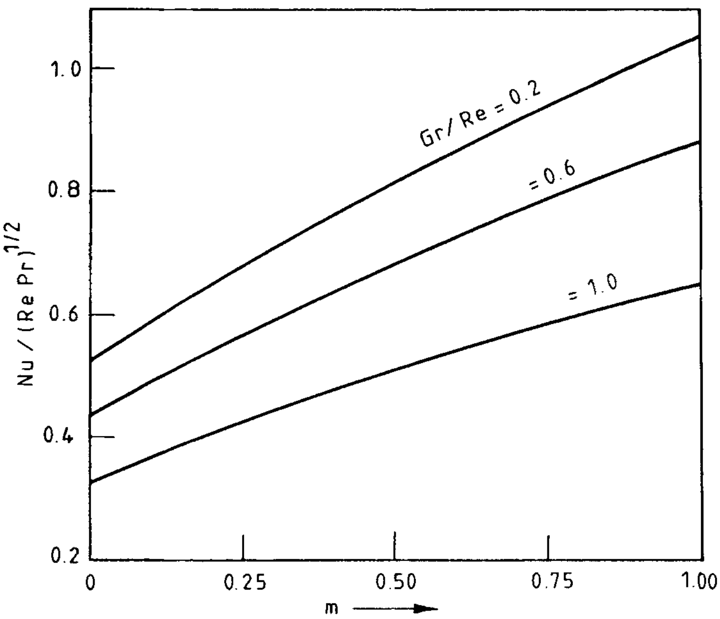


Figure 8. Effect of $m$ or $Nu/(Re\,Pr)^{1/2}$, opposing flow

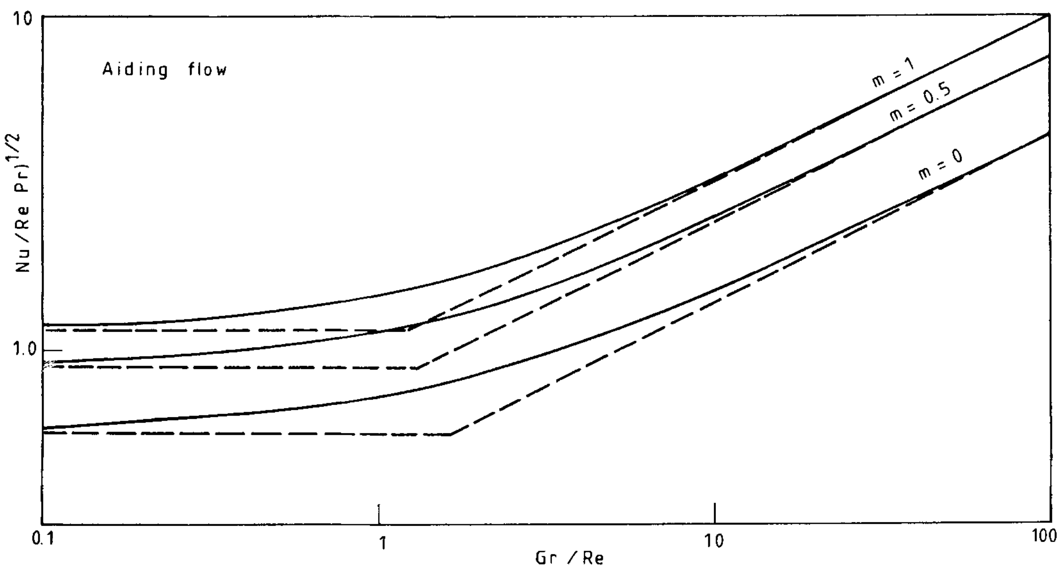


Figure 9. Heat transfer results; aiding flow

value of $Gr/Re$ as $m$ increases, as can be seen from Figure 9. In the case of opposing flows it is seen from Figure 10 that the heat transfer rate decreases as $Gr/Re$ increases. For a given value of $Gr/Re$ the reduction in heat transfer is greater for higher values of $m$ (especially when $Gr/Re > 0{\cdot}7$).

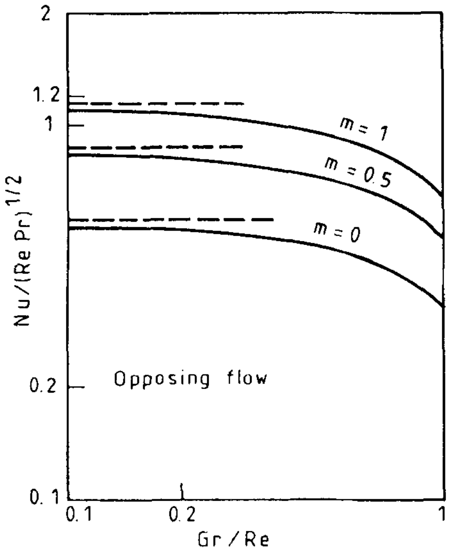


Figure 10. Heat transfer results; opposing flow

## 4. CONCLUSIONS

The present paper deals with the determination of the heat transfer rate for combined free and forced convection in an external flow through fluid-saturated porous media adjacent to a non-isothermal (arbitrarily varying) vertical wall. Both aiding and opposing flows have been analysed. A wall temperature variation of the power-law type $T_w - T_\infty = M_y^m$ is shown to be a particular case and solutions are given for various value of the index $m$. The solutions are obtained once and for all by using the universal functions derived in the analysis (whereas earlier authors had to solve for each value of $m$ to get the solution). The results have been compared with those obtained by a similarity analysis and the agreement is found to be good. The universal constants can also be used to obtain solutions for any other type of wall temperature variation.

## NOMENCLATURE

| | |
|---|---|
| $f$ | non-dimensional stream function (equation (15)) |
| $F, f_0 f_1,$ $f_{00}$, etc. | universal functions for velocity (equation (23)) |
| $g$ | acceleration due to gravity |
| $Gr$ | Grashof number (equation (20)) |
| $H$ | universal function for temperature (equation (24)) |
| $h$ | local heat transfer coefficient (equation (33)) |
| $k_m$ | equivalent thermal conductivity of saturated porous medium |
| $K$ | permeability of the medium |
| $Nu$ | Nusselt number (equation (34)) |
| $n$ | integer constant (equation (22)) |
| $p$ | pressure |
| $q$ | heat flux (equation (31)) |

| | |
|---|---|
| $Re$ | Reynolds number (equation (21)) |
| $T$ | temperature |
| $u$ | area-averaged velocity in $x$-direction |
| $v$ | area-averaged velocity in $y$-direction |
| $V_\infty$ | area-averaged free stream velocity |
| $x$, $y$ | co-ordinate system shown in Figures 1(a) and (b) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | effective thermal diffusivity in porous medium |
| $\beta$ | coefficient of thermal expansion |
| $\eta$ | dimensionless variable (equation (17)) |
| $\theta$ | dimensionless temperature (equation (16)) |
| $\{\lambda_n(y)\}$ | a set of functions of $y$ (equation (22)) |
| $\mu$ | dynamic viscosity of the fluid |
| $\nu$ | kinematic viscosity of the fluid |
| $\rho$ | fluid density |
| $\psi$ | stream function (equation (8)) |

*Subscripts*

| | |
|---|---|
| w | for wall conditions |
| $\infty$ | for condition outside the boundary layer |

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